

**AD-A274 599**



7

**FINAL REPORT TO ONR**

**CONTRACT NO. N00014-91J-1187, CODE 1132-SM**

**NEW VARIATIONAL TECHNIQUES FOR ACOUSTIC RADIATION**

**AND SCATTERING FROM ELASTIC SHELL STRUCTURES**

**PRINCIPAL INVESTIGATOR: JERRY H. GINSBERG**

**SCHOOL OF MECHANICAL ENGINEERING**

**GEORGIA INSTITUTE OF TECHNOLOGY**

*12-20-93*

**DTIC**  
**ELECTE**  
**JAN 10 1994**  
**A**

This document has been approved  
for public release and sale; its  
distribution is unlimited.

98

**93-31610**



CONTRACTING SUPPORT DIVISION  
404•894•6944

Telex: 542507 GTRC OCA ATL  
Fax: 404•894•5945

December 20, 1993

REPLY TO: E-25-634

Office of Naval Research  
ATTN: Dr. Phillip B. Abraham  
Code 1132SM  
800 North Quincy Street  
Arlington, VA 22217-5000

SUBJECT:

Final Technical Report  
Project Director: J.H. Ginsberg  
Contract No.: N00014-91-J-1187  
"NEW VARIATIONAL ACOUSTIC RADIATION..SCATTERING ELASTIC  
SHELL STRU"  
Period Covered: 901001-930930

The subject report is forwarded in conformance with the contract/grant specifications.

Should you have any questions or comments regarding this report, please contact the Project Director or the undersigned at 404/894-4764.

Sincerely,



/arl

Wanda Simon  
Research Reports Coordinator

Distribution:  
Addressee, 3 copies  
CODE 2627, 1 copy  
Defense Technical Information Center, 4 copies  
cc: ONR-RR

# ONR FINAL REPORT CONTENTS

I.	OVERVIEW OF PROJECT ACTIVITIES .....	3
II.	BASIC PRINCIPLE .....	5
	1. General Implementation of SVP for Bodies of Revolution.....	7
	2. Structural Dynamics for Plates and Shells .....	12
	3. Modifications for Scattering .....	14
III.	RESEARCH TOPICS AND RESULTS .....	15
	1. Radiation from Vibrating Rigid Bodies .....	15
	2. Radiation from Elastic Bodies .....	17
	3. Scattering from Rigid Bodies .....	18
	4. The Interior Cavity Problem .....	18
	5. Eigenvalue Veering Phenomena .....	19
IV.	CLOSURE .....	20
V.	PAPERS, THESES, AND REPORTS .....	21
VI.	PROJECT PARTICIPANTS .....	26

DTEG QUALITY INSPECTED 8

Accession For	
NTIS	CRA&I
DTIC	TAB
Unannounced	
Justification	
By	
Distribution/	
Availability Code	
Dist	Avail and/or Special
A-1	

## I. OVERVIEW OF PROJECT ACTIVITIES

The interaction between a vibrating submerged structure and the surrounding fluid, which features coupling between the surface pressure distribution and the structural displacement, is an inherent feature for sound radiation and target strength analyses. A variety of approaches have been implemented in the past, but each suffers from serious limitations. Formal mathematical analysis using separation of variables or integral transform techniques is suitable only for the simplest structural models, while full finite element descriptions of realistic structures and the surrounding medium lead to excessively large computer simulations. One approach uses approximate impedance-type boundary condition of uncertain accuracy to model the fluid response. Boundary element formulations rationally represent the interaction phenomena without explicitly solving field equations for the fluid, at the expense of an enormous increase in computational effort due to the need to cover the surface with a reasonably fine mesh.

Because of the limitation of classical analytical techniques, it was widely believed that analytical-type solutions can only be obtained in idealized systems. The primary objective of the project was to develop an analytical-type approach for modeling fluid-structure interaction in the frequency domain. This objective was achieved by developing the surface variational acoustics principle (SVP). SVP represents the spatial dependence of the surface response as a series expansion in a set of assumed basis functions. Since the solution derived from SVP is the coefficients of this series, the primary difference from the results of a classical analysis lies in the fact that SVP determines these coefficients by numerical techniques. In essence, implementation of SVP is comparable to using the method of assumed modes in structural dynamics. When the mechanical energy functions are very complicated, as in the case of a doubly curved-shell, determination of the system coefficients for the latter method also requires numerical techniques to evaluate the stiffness and inertia coefficients. A specific advantage of SVP formulations in comparison to conventional finite element and boundary element formulations is the fact that convergence of the representation can be judged by examining the solution. One merely needs to

identify that the higher order modal amplitudes are small -- there is no need to construct a more refined solution to use as a reference for convergence. In all of the problems addressed by SVP, convergence was found to be quite rapid. Furthermore, even when a result was not fully convergent, it was found nevertheless to be remarkably close to the convergent result.

Because SVP represents the surface pressure in a functional series form, it involves a substantially reduced set of unknowns in comparison to boundary element formulations, which are based on point-wise discretizations. In addition, SVP is essentially an optimization process that selects the coefficients of the modal series so as to minimize the deviation of the derived solution relative to the true one.

The objective of the first phase of the project was to develop the SVP in situations where the surface motion is specified. Such a case arises when a rigid body is set into a specified motion. Meeting this objective entailed validating SVP results against known analytical solutions. The next research phase addressed the broad objective of extending SVP to treat acoustic radiation from elastic structures. This objective was achieved by coupling the SVP equations to the method of assumed modes for the structural vibration. The latter method was chosen for the description of the structural motion because it too is variationally based, being founded on Hamilton's principle. In the first two research phases, the analysis was restricted to cases of axisymmetric excitation. The next phase met the objective of generalizing SVP to treat situations that are not axisymmetric. Two cases were considered: nonsymmetric excitation of bodies of revolution and two-dimensional (plane strain) models. The final objective was to extend SVP to treat scattering of waves at arbitrary incidence on an elastic body.

Two considerations influenced the selection of problems for each research phase. Problem that had been solved previously provided validation of SVP. Once the formulation was validated, problems were selected on the basis of their relevance to specific structural acoustic phenomena. A particular problem area that received much attention was the influence of eigenvalue veering phenomena on predictions of acoustical performance. Such

phenomena arise when alteration of a system parameter causes the natural frequencies of different modes to become similar. The in-vacuo mode shapes in this case are highly sensitive to the value of the system parameter, which suggests that the forced response of the submerged system should display similar sensitivity.

## II. BASIC PRINCIPLE

The derivation of the variational principle is fairly straightforward. The starting point is the surface integral equation for normal velocity. The steps involved in deriving this equation begin by taking the gradient at a field point of the Kirchhoff-Helmholtz integral theorem. Before the field point is brought to the surface, a regularization procedure is used to remove a hypersingularity. This regularization is needed because one would otherwise need to integrate over a double gradient of the Green's function. The equation derived in this manner is called the *second surface integral equation*. The field points and source points appear in symmetrical form in this integral equation, so it is imminently suitable for constructing a variational principle. The SVP theorem is obtained by multiplying the second surface integral equation by a virtual increment in the surface pressure at a surface field point, and then integrating over all such points on the surface. Once one accounts for the symmetry of the free-space Green's function between the field and source points, it becomes obvious that the integration over the surface yields an expression that is the first variation of a functional  $J[p]$ , with the surface velocity held constant. The theorem states that the value of a functional  $J$  that depends on the pressure field is stationary to infinitesimal variations of that field when  $p$  is the true pressure. The specific forms are

$$\delta J = 0 \tag{1}$$

where

$$\begin{aligned}
J[p] = & \frac{1}{2} \iint_S \iint_S \left\{ k^2 [ \vec{n}(\vec{r}) \cdot \vec{n}(\vec{\xi}) ] p(\vec{r}) p(\vec{\xi}) \right. \\
& - [ \vec{n}(\vec{r}) \times \nabla_{\vec{r}} p(\vec{r}) ] \cdot [ \vec{n}(\vec{\xi}) \times \nabla_{\vec{\xi}} p(\vec{\xi}) ] \left. \right\} G(\vec{r}, \vec{\xi}) dA_{\vec{\xi}} dA_{\vec{r}} \\
& - 4\pi i \omega \rho \iint_S p(\vec{r}) U_n(\vec{r}) dA_{\vec{r}} \quad (2)
\end{aligned}$$

Note that the variational principle governs the surface pressure corresponding to a specified velocity field, so the normal velocity  $\vec{v}_n$  is held constant in the variation. The quantity  $U_n(\vec{r})$  is a functional of the surface velocity whose evaluation entails integrating over all points on the surface, specifically

$$U_n(\vec{r}) = \frac{1}{2} \vec{v}_n(\vec{r}) + \frac{1}{4\pi} \text{PR} \left\{ \iint_S \vec{v}_n(\vec{\xi}) \left[ \vec{n}(\vec{r}) \cdot \frac{\vec{r} - \vec{\xi}}{ar} \right] \right\} \frac{d}{dr} G(\vec{r}, \vec{\xi}) dA_{\vec{\xi}} \quad (3)$$

In these expressions,  $ar$  is the distance between the field and source point,  $ar = \vec{r} - \vec{\xi}$ , and  $G(\vec{r}, \vec{\xi})$  denotes the nondimensional free-space Green's function,

$$G(\vec{r}, \vec{\xi}) = \begin{cases} \frac{1}{r} \exp(ikar): \text{three-dimensions} \\ \frac{1}{4i} H_0^{(1)}(kar): \text{two-dimensions} \end{cases} \quad (4)$$

Also, PR denotes the Cauchy principle value of the integral, which is obtained by excluding from the region of integration an infinitesimal patch centered on  $\vec{r} = \vec{\xi}$ .

Certain features of the expression for  $J[p]$ , Eq. (2), should be noted. All gradients of pressure at any point on the surface are crossed with the normal vector at that point, so the derivatives represent the gradient of the pressure tangentially to the surface. No gradients of the Green's function appear in the pressure integrals, but the normal velocity functional  $U_n(\vec{r})$  does contain such a gradient. This term actually has the same singularity as the Green's function, because the dot product of  $\vec{n}(\vec{r})$  and  $\vec{r} - \vec{\xi}$  behaves as  $r^2$  as  $\vec{\xi} \rightarrow \vec{r}$ . As a consequence, all singularities arising in the evaluation of  $G(\vec{r}, \vec{\xi})$  are integrable.

In general, a variational principle has two uses. One can apply the calculus of variations to obtain the governing equations for the system. Such an equation is known as the Euler-Lagrange equation for the variational principle. Not surprisingly, the Euler-Lagrange equation obtained from Eqs. (1) and (2) is the second surface integral equation. A variational principle may also be used as the basis for an approximate solution. This is the way in which it was used in the project.

## II.1 General Implementation of SVP for Bodies of Revolution

The latest implementation of SVP considers an arbitrary excitation of a body of revolution. The shape of the body is specified by describing the cylindrical coordinate functions of its shape generator. This dependence is expressed in parametric form, with the parameter selected as the arclength  $s$  measured along the generator from one apex. It is convenient to work with nondimensional variables. For this reason, the parameter is selected as  $\alpha = s/a$ , where the reference length  $a$  is defined as the largest radius. Thus, the two variables locating a point on the surface are the nondimensional meridional arclength  $\alpha$  and the azimuthal angle  $\theta$ , with the axial and transverse distance for this point given by specifying  $aR(\alpha)$  and  $az(\alpha)$ . Evaluation of the SVP integrals requires definition of the second surface point  $\bar{\zeta}$  which is specified by its arclength parameter  $\beta$  and an azimuthal angle  $\theta$  relative to the first point.

The arclength and azimuthal angle locating a point on the surface are also used to describe the surface pressure and surface velocity. The  $2\pi$  periodicity of the azimuthal dependence enables one to expand  $p$  and  $v_n$  in Fourier series. Using the complex form of such series simplifies some of the ensuing steps, so the pressure and normal velocity distributions on the surface are described by

$$\begin{aligned} p(\bar{\zeta}, r) &= \frac{1}{2} \rho c^2 P(\alpha, \theta) \exp(i\omega t) + \text{C.C.} \\ v_n(\bar{\zeta}, r) &= \frac{1}{2} c T(\alpha, \theta) \exp(i\omega t) + \text{C.C.} \end{aligned} \quad (5)$$



where C.C. denotes the complex conjugate of preceding terms, and the spatial distributions are

$$P = \frac{1}{2} \sum_{m=-\infty}^{\infty} P_m(\alpha) \exp(im\theta) \quad T = \frac{1}{2} \sum_{m=-\infty}^{\infty} T_m(\alpha) \exp(im\theta) \quad (6)$$

These expressions and similar ones for the second surface point are substituted into the definition of  $J[p]$ , Eq. (2). This enables one to carry out the azimuthal portion of each surface integral. As a result of the axial symmetry of the shape, the azimuthal harmonics uncouple. The variational functional is thereby found to have the form

$$J = \rho^2 c^4 a \sum_{m=-\infty}^{\infty} J_m[P_m] \quad (7)$$

where the nondimensional functional for each azimuthal harmonic is given by

$$\begin{aligned} J_m = \frac{\pi}{4} \int_0^l \int_0^l \left\{ C_m(\alpha, \beta) [k^2 a^2 R(\alpha) R(\beta) R'(\alpha) R'(\beta) - m^2 z(\alpha) z(\beta)] P_m(\alpha) P_{-m}(\beta) \right. \\ + D_m(\alpha, \beta) [k^2 a^2 R(\alpha) R(\beta) z'(\alpha) z'(\beta) - m^2 R'(\alpha) R'(\beta)] P_m(\alpha) P_{-m}(\beta) \\ - R(\alpha) R(\beta) P'_m(\alpha) P'_{-m}(\beta) \left. \right\} - E_m(\alpha, \beta) m [R'(\alpha) R(\beta) P_m(\alpha) P'_{-m}(\beta) \\ + R(\alpha) R'(\beta) P'_m(\alpha) P_{-m}(\beta)] \Big\} d\alpha d\beta - \pi^2 k^2 a^2 \int_0^l R(\alpha) T_m(\alpha) P_{-m}(\alpha) d\alpha \\ - \frac{\pi^2}{2} k^2 a^2 \int_0^l \int_0^l F_m(\alpha, \beta) T_m(\alpha) P_{-m}(\beta) d\alpha d\beta \quad (8) \end{aligned}$$

In the foregoing a prime indicates differentiation with respect to the argument, and  $l = \max(s)/a$ .

The functions  $C_m(\alpha, \beta)$ ,  $D_m(\alpha, \beta)$ ,  $E_m(\alpha, \beta)$ , and  $F_m(\alpha, \beta)$  are integrals over the azimuthal angle of the Green's function, weighted by various factors. Only the first and last of these functions need to be computed. The others are obtainable from recursive identities.

$$C_m(\alpha, \beta) = 2 \int_0^\pi \frac{1}{r} \exp(ikar) \cos(m\theta) d\theta \quad (9)$$

$$D_m(\alpha, \beta) = \begin{cases} \frac{1}{2}[C_{m-1}(\alpha, \beta) + C_{m+1}(\alpha, \beta)] & \text{if } m > 0 \\ C_1(\alpha, \beta) & \text{if } m = 0 \end{cases} \quad (10)$$

$$E_m(\alpha, \beta) = \begin{cases} \frac{1}{2}[C_{m-1}(\alpha, \beta) - C_{m+1}(\alpha, \beta)] & \text{if } m > 0 \\ 0 & \text{if } m = 0 \end{cases} \quad (11)$$

$$F_m(\alpha, \beta) = 2R(\alpha)R(\beta) \int_0^\pi \left\{ z'(\alpha)[R(\alpha) - R(\beta) \cos \theta] + R'(\alpha)[z(\alpha) - z(\beta)] \right\} \frac{ikar - 1}{r^3} \exp(ikar) \cos(m\theta) d\theta \quad (12)$$

where the distance  $r$  between points on the surface is related to the shape functions by

$$r = \left\{ R(\alpha)^2 + R(\beta)^2 - 2R(\alpha)R(\beta)\cos \theta + [z(\alpha) - z(\beta)]^2 \right\}^{1/2} \quad (13)$$

Each integrand appearing in Eqs. (9) and (12) contains a  $1/r$  singularity, whose contribution is isolated as elliptic integrals. The remainders are computed by conventional numerical methods.

The next step in the formulation of SVP is to express the unknown surface pressure as Ritz series expansion, whose general form is

$$P_m(\alpha) = \sum_{j=1}^N P_{mj} \psi_{mj}(\alpha) \quad (14)$$

where the coefficients  $P_{mj}$  are the quantities to be determined. The conditions imposed on the  $N$  basis functions  $\psi_{mj}$  are minimal. Necessary conditions are that reconstruction of the pressure distribution be continuous everywhere, including the boundaries  $\alpha = 0$  and  $\alpha = \ell$ , which are the apexes of the body of revolution. Other conditions, such as symmetry about any of the coordinate planes, are equivalent to natural boundary conditions in structural dynamics. If such conditions are not inherent to the basis

functions selected for the analysis, they will nevertheless be exhibited by the SVP solution.

In the initial phases of the project, where SVP was verified against classical analytical results, *a priori* knowledge of the solution was usually used to select the appropriate basis functions. With the ultimate merger of SVP with structural dynamics, such knowledge was not available. The wave-number based formulation was developed to address this situation. Because the arclength  $\max(s)$  is the extent over which waves may travel, it represents their effective aperture. Correspondingly, all waves must be representable as half-range Fourier series using either cosines or sines. Which type of function one should use depends on the azimuthal harmonic number. For  $m = 0$ , the axisymmetry condition at the apexes is satisfied by cosines, while continuity of pressure at those locations requires the usage of sines for  $n > 0$ . Thus, the basis functions are selected as

$$\psi_{mj} = \psi_{-mj} = \begin{cases} \cos(j\pi\alpha) & \text{if } m = 0 \\ \sin(j\pi\alpha) & \text{if } m > 0 \end{cases} \quad (15)$$

The reason for referring to this formulation as the wave-number based SVP comes from replacing the trigonometric functions above with complex exponentials, and then combining the two Fourier series for meridional and azimuthal dependence of pressure. The form of this combination is

$$p(\bar{r}, t) = \sum_{m=-\infty}^{\infty} \sum_{j=1}^N P_{mj} \exp \left[ i \left( m\theta \pm \frac{j\pi s}{\max(s)} - \omega t \right) \right] + C.C. \quad (16)$$

which shows that the  $P_{mj}$  coefficients are the amplitudes of the various pressure waves whose rays trace spirals along the wetted surface. Thus, the results obtained from this formulation of SVP are the analytical analogs of the experimental data ultimately provided by acoustical holography.

The final step in the formulation of SVP involves evaluating the various functionals  $J_m$ , Eq. (8), resulting from substitution of the series expansion for surface pressure, Eq. (14). The general form of this result is

$$J_m = \frac{1}{2} \sum_{j=1}^N \sum_{n=1}^N A_{mjn} P_{mj} P_{-mn} - k^2 a^2 \sum_{j=1}^N B_{mj} P_{-mj} \quad (17)$$

The coefficients  $A_{mjn}$  and  $B_{mj}$  are obtained from double integrals expressing all possible position pairs along a meridian. Their definition is the same as the corresponding integrals in Eq. (8) above, with  $P_m(\alpha)$  replaced by  $\psi_{mj}(\alpha)$ , etc. Because the domain of integration for both sets of coefficients extends over the singularity at  $\alpha = \beta$ , the domain is broken down into squares and triangles whose sides are parallel to that diagonal. The numerical values are obtained by application of a Gaussian rule based on nine interior points for each subdomain.

Because Eq. (17) describes  $J_m$  as a function of the  $P_{mj}$  coefficients, the first variation is

$$\delta J_m = \sum_{j=1}^N \frac{\partial J_m}{\partial P_{-mj}} \delta P_{-mj} = 0 \quad (18)$$

This relation must hold for arbitrary  $\delta P_{-mj}$ , which can only be true if all partial derivatives vanish independently. The matrix form of these derivatives is

$$[A_m](P_m) = k^2 a^2 (B_m) \quad (19)$$

where  $(P_m)$  contains the  $N$  values of  $P_{mj}$ .

Evaluation of the coefficients  $B_{mj}$  requires a description of the manner in which the surface normal velocity depends on the meridional arclength for each azimuthal harmonic. In the case of radiation due to rigid body motion, such dependence is readily obtained by taking the component of the rigid body velocity in the direction normal to each point on the surface. Evaluation of radiation from elastic bodies requires structural dynamics considerations. (Acoustic scattering is treated by modifying the formulation for radiation in a manner that will be discussed later.)

## II.2. Structural Dynamics for Plates and Shells

Many methods for analyzing structural motion are available, including finite elements, finite differences, and boundary elements. The one selected to be combined with SVP was the method of assumed modes, which more properly should be referred to as a Ritz expansion. This method is closest to SVP from a conceptual standpoint, because both represent the response over the entire domain in terms of series of global basis functions. (It would not be difficult to modify the formulation to accommodate the local basis functions associated with a finite element description of the structure.) The deformation of a plate or shell is described by displacement component  $w$  normal to its midsurface, and in-plane displacements,  $u$  and  $v$ . For a body of revolution these components are in the meridional and azimuthal directions. Each displacement component is expanded in a series comparable to that used to describe surface pressure,

$$\begin{aligned} u &= \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{j=1}^N U_{mj}(t) \psi_{mj}^u(\alpha) \exp(im\theta) \\ v &= \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{j=1}^N V_{mj}(t) \psi_{mj}^v(\alpha) \exp(im\theta) \\ w &= \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{j=1}^N W_{mj}(t) \psi_{mj}^w(\alpha) \exp(im\theta) \end{aligned} \quad (20)$$

where the various  $\psi_{mj}^{(\cdot)}$  are admissible basis functions for the displacement component indicated by the superscript. (Admissible functions are those that satisfy the geometric boundary conditions and have the appropriate degree of differentiability.)

The equations of motion governing the amplitudes of the basis functions are obtained by using the series to construct the kinetic and strain energy. The virtual work must also be described in terms of these displacements, with one term consisting of the integral of  $(-p \delta w)$  to describe the effect of the acoustic pressure. This procedure leads to the the generalized

forces  $Q^{(i)}$  associated with each displacement component. Hamilton's extended principle for this system reduces to Lagrange's equations for the basis function amplitudes. Redefining these amplitudes by factoring out the harmonic temporal variation leads to

$$\left[ \begin{bmatrix} K_{uu} & K_{uv} & K_{uw} \\ K_{uv}^T & K_{vv} & K_{vw} \\ K_{uw}^T & K_{vw}^T & K_{ww} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{uu} & 0 & 0 \\ 0 & M_{vv} & 0 \\ 0 & 0 & M_{ww} \end{bmatrix} \right] \begin{Bmatrix} U_m \\ V_m \\ W_m \end{Bmatrix} = \begin{Bmatrix} Q_u \\ Q_v \\ Q_w - \Gamma P_m \end{Bmatrix} \quad (21)$$

where each element in the above matrices represents an  $N$  dimensional partition. The various  $K$  and  $M$  represent stiffness and mass coefficients associated with the displacement components indicated by their subscript, the  $Q$  terms are the generalized forces due to the dynamic excitation, and  $\Gamma$  are the coupling factors describing the contribution of each pressure basis function to the generalized force.

Because the normal components of velocity in the fluid and the structure must be continuous across the wetted surface, the characterization of the normal displacement as a series defines the  $T_m$  terms in Eq (6), which is the meridional dependence of normal velocity in the  $m$  th azimuthal harmonic. Matching the meridional dependencies of the structure and fluid velocities leads to

$$T_m = -ika \sum_{j=1}^N W_{mj}(t) \psi_{mj}^w(\alpha) \quad (22)$$

When this representation of the meridional variation of the  $m$  th harmonic of surface velocity is used to evaluate the coefficients  $B_{mj}$  in Eq. (17), one finds

$$B_{mj} = \sum_{n=1}^N \Lambda_{mjn} W_{mn} \quad (23)$$

where  $\Lambda_{mjn}$  represent coupling factors describing the mapping of the structural velocity into the functional space used to describe the pressure. The result of using the foregoing to form the SVP equations is

$$[\Lambda_m](P_m) - k_a^2 [\Lambda_m](w_m) \quad (24)$$

The coupled response of the surface pressure and the motion of the structure in each azimuthal harmonic may be determined solving the SVP equation (24) simultaneously with the structural dynamics equation (21).

### II.3. Modifications for Scattering

The development thus far addressed radiation problems. A simple modification extends the treatment to cases where the excitation is an incident acoustic signal. The key is to decompose the pressure into incident and scattered parts. Then the scattered pressure on the surface satisfies the same set of surface integral equations as the total surface pressure in a radiation problem, provided that the normal velocity is defined as the particle velocity in the scattered signal. Continuity of normal velocity on the surface requires that this particle velocity is the difference between the structure's velocity and the normal component of particle velocity in the incident wave. Euler's equation gives the latter, so that

$$\bar{n} \cdot \nabla p^{sca} = k_a^2 w - \bar{n} \cdot \nabla p^{inc} \quad (25)$$

It follows that replacing any term traceable to  $\bar{n} \cdot \nabla p$  in the SVP equations by the right side of the above leads to relations governing the scattered part of the pressure field.

In order to apply this result, the incident pressure and particle velocity are expanded in azimuthal Fourier series.

$$p^{inc}(\vec{r}) = \frac{1}{2} \rho c^2 \sum_{m=-\infty}^{\infty} P_m^{inc}(\alpha) \exp(im\theta)$$

$$\vec{n}(\vec{r}) \cdot \nabla_{\vec{r}} p^{inc}(\vec{r}) = \frac{1}{2} \rho c^2 \sum_{m=-\infty}^{\infty} T_m^{inc}(\alpha) \exp(im\theta) \quad (26)$$

This converts the SVP equations to

$$[A_m](P_m) - k^2 a^2 [A_m](w_m) = k^2 a^2 (B_m^{inc})$$

where  $(B_m^{inc})$  represents the values of  $B_m$  corresponding to a surface velocity whose meridional dependence is  $T_m^{inc}(\alpha)$ . In addition, the presence of an incident pressure field adds to the generalized forces driving the normal displacement. This leads to an additional generalized force, which is described by replacing  $Q_w$  by  $Q_w + Q_w^{inc}$ . The solution of the SVP and structural dynamics equations in this approach yield the displacement amplitudes and the scattered part of the pressure field. The total surface pressure may then be determined by superposing the incident field's contribution to each azimuthal harmonic.

### III. RESEARCH TOPICS AND RESULTS

The following discusses the themes of the various research phases of the project. The numbers appearing at the end of each section refer to the papers listed in the next section.

#### III.1. Radiation from Vibrating Rigid Bodies (papers # 1-8)

Each research phase in which the SVP approach was generalized began with the task of evaluating the surface pressure on a body whose motion is prescribed. The first cases to be pursued were a transversely oscillating thin rigid disk, a sphere whose surface is executing a breathing mode vibration, and a rigid sphere and a flat-ended cylinder oscillating parallel to the axis of symmetry. The basis functions for pressure were initially suggested by the available analytical solutions, rather than the sinusoidal



forms associated with the wavenumber formulation of SVP. A study of the oscillating rigid disk was also carried out using elemental basis functions comparable to those employed for boundary elements.

Each of the foregoing cases constitutes an axisymmetric system. The extension of SVP to nonsymmetric cases addressed a hemi-capped cylinder, and a spheroid undergoing rigid body motion in the axial and beamwise direction, as well as a rotational oscillation about the centroid. The former problem provided an important consistency check. If one sets the length of the cylindrical section to zero, the system is a sphere. The pressure distribution for beamwise translation is not axisymmetric, whereas axial translation represents an axisymmetric normal velocity distribution. It was verified that the SVP results for the beamwise case gave the same surface pressure as that obtained for axial motion, and both results were verified to give the same result as the analytical solution for the axial translation. It was in this phase of the research program that the wavenumber formulation of SVP was developed.

Addressing problems of this type was very useful for a number of reasons. Because structural dynamics equations were not an issue, it was easier to debug the implementation of the SVP equations. Furthermore, rigid body motions excite only the  $m = 0$  and  $m = 1$  azimuthal harmonics, so it was not necessary to worry about convergence and numerical error associated with high harmonics. Another aid to validating the analysis was the fact that analytical solutions were available in some cases. In addition, studies of slender cylinders and prolate spheroids led to some interesting results regarding the effects of curvature on a surface. In every case, convergence of the pressure amplitudes led to results that differed by less than 1% from those of analytical solutions. Similar close agreement for a hemi-capped cylinder was obtained from a comparison with the predictions of the CHIEF boundary element program. It is important to realize that convergence of the series occurs only if the higher order terms in series are very small. It follows that one can examine the magnitude of the higher order terms to determine that the solution has converged--there is no need to perform an analysis using a different series length to determine the adequacy of a result.

### III.2. Radiation from Elastic Bodies (papers # 9-22)

The research plan was to use the rigid body motion problem to generalize the SVP formulation. Once each effort was finished, the next task was to implement the structural dynamics aspects for that version. Thus, the first case where structural dynamics was considered was a circular elastic membrane or flexural disk whose boundary is supported by an annular baffle. The width of this baffle could vary from zero to infinity. Only axisymmetric excitations were considered. The basis functions for the structure in these studies were selected as the *in-vacuo* normal modes. Several types of basis functions for the surface pressure representation were addressed. It was shown that the basis functions used to study the rigid body case lead to ill-conditioning when a large number of functions are retained, because the high order functions are not linearly independent from a computational viewpoint. Bessel and sinusoidal basis functions were shown to agree well with the analytical solution derived by Alper and Magrab, as were boundary element-type functions that spanned small subdomains. A further study of the elastic plate considered the effect of annular stiffeners. This problem introduced the concept of component mode analysis, which was subsequently implemented by Igusa and Achenbach of Northwestern University as a fundamental aspect of their ONR sponsored research program.

The next elastic structure to be addressed was a spheroidal shell of arbitrary aspect ratio. Two formulations of the fluid-structure coupling were derived. The first, which was termed the "direct method" followed the formulation described in the previous section. The second was called the "modal method," because the assumed modes method is used to evaluate the *in-vacuo* natural frequencies and mode shapes. These free vibration properties were then used as the basis functions for the SVP formulation. The results for free vibration were shown to be extremely accurate, and to be more comprehensive than any previously available. An interesting feature was shown to be a coalescence of eigenvalues at certain aspect ratios, which launched a separate research phase that will be discussed later. The only data available for validation in the fluid-loaded problem was for a spherical shell. The SVP results were shown to be very close to Hayek's analytical solution.

An elastic plate in an annular baffle was the first study to remove the restriction to axisymmetric situations. The plate was subjected to an eccentric concentrated harmonic force normal to the surface. This study was the first to implement the Fourier series representation of the azimuthal dependence. No prior work had considered this problem, but the results were examined for self-consistency and physical reasonableness. At the present time, the corresponding SVP implementation for shells is under development.

### III.3. Scattering from Rigid Bodies (paper # 23)

It was decided to delay considering acoustic scattering until the SVP formulation was generalized to handle non-axisymmetric problems. The incident signal in each SVP analysis was a plane acoustic wave at arbitrary incidence. Results have been obtained for scattering from flat and hemi-capped cylinders, and from prolate spheroids whose surface is stationary. These were compared to numerical solutions obtained from the CHIEF and non-symmetric SHIP programs. In every case the results were virtually identical to the numerical solutions. A consistency check was also carried out for the case of a sphere, where the solution should yield the same pressure distribution as that predicted analytically, independent of the angle of incidence.

### III.4. The Interior Cavity Problem (papers # 21, 22, 24, 25)

Because the stationary principle for an SVP formulation is derived from the second surface integral equation, there is no unique solution at frequencies that match the eigenfrequencies for the interior Neumann problem (zero velocity on the boundary). A study was performed to assess whether the over-determined formulation employed in CHIEF could be used. This entailed adding auxiliary constraint equations requiring that the interior pressure vanish at selected points or on average over an interior surface. Although the concept worked well, it has not been implemented into the general SVP formulation. This decision stemmed from observations in the analysis of radiation from a spheroidal shell. That work showed that even if one were to come very close to one of these frequencies, the effect would be either very minor or imperceptible. The apparent reason for this result is the limited size of an SVP model, i.e. the relatively small number of terms required to obtain an accurate solution. As a consequence, the system

of equations remains well-conditioned even if one is within 0.01% of the spurious resonance frequency. This is an important factor because the decomposition of the problem into azimuthal harmonics means that the spurious resonance frequencies will occur in relatively equal density at each azimuthal harmonic. In contrast, conventional boundary element formulations encounter an increasing density of spurious frequencies as the excitation frequency increases, because they treat all azimuthal harmonics simultaneously.

### III.5. Eigenvalue Veering Phenomena (papers # 26-33)

The analysis of the *in-vacuo* modes of a spheroidal shell disclosed that at certain length to radius ratios, the natural frequencies of the flexural and extensional branches become very close. Further increase of this aspect ratio led to modal vibration patterns that exhibited localized displacement patterns. Comparable phenomena, which are grouped under the term of "eigenvalue veering phenomena", had been studied for other systems. However, all such work failed to describe the actual extreme sensitivity of the mode shapes as the system parameter is varied through the zone where eigenvalue veering occurs. A general treatment was derived as part of the research program, and then applied to the spheroidal shell problem. It was shown that the localized modes cannot be omitted from the modal version of SVP, even if their natural frequencies are much higher than the excitation frequency. This result apparently stems from the extreme efficiency with which these modes radiate.

Another study of the effect of eigenvalue veering phenomena on acoustic radiation was carried out using a two-dimensional flat plate in an infinite baffle. Addressing this situation necessitated extending SVP to handle two-dimensional (plane strain) problems. That formulation was validated by comparing the SVP results for a single-span plate to the integral transform solution. Then the formulation was applied to a two-span plate whose individual spans are nearly uncoupled by a stiff torsional spring acting at the center support. The *in-vacuo* analog of this problem had served as a prototype for more than a decade for studies of eigenvalue veering. The startling result that emerged from this work was that a shifting the position of the center support by as little as 1% of the span length can raise

the total radiated power by a factor of two. Because a complicated system such as submarine is sure to have natural frequencies that are very close, it is reasonable to anticipate that comparable eigenvalue veering phenomena will occur in those systems. This has significant implications for constructing mathematical models, because it suggests that little details of construction may have a strong influence on acoustical performance.

#### IV. CLOSURE

This research project has resulted in development of the SVP technique for modeling the coupled acoustic-structure response of submerged bodies. The technique requires determination of a comparatively small number of variables. In the wavenumber version of SVP, these variable are particularly meaningful from a physical standpoint, because they represent the amplitudes of helical-type waves that propagate through the structure and through the fluid. SVP solutions have the important property of allowing one to ascertain that the results are convergent and represent the response to a satisfactory level, without requiring evaluation of a more refined model for comparison. A number of generalizations ultimately yielded a technique that is equally suitable for radiation and scattering. The only capability requiring further development is analysis of non-axisymmetric problems in curved shell structures. Because of the relatively small number of unknowns to be determined, and the fact that those unknowns are physically meaningful, the technique provides physical insight into the response of submerged structures. These features make SVP very useful as a research tool, as well as a tool for design parameter studies.

## V. PAPERS, THESES, AND REPORTS

- (1) J. H. Ginsberg, A. D. Pierce, X.-F. Wu, and J. S. DiMarco, "Rayleigh-Ritz Description Derived from Variational Principles of Diffraction by a Circular Disk," 109th Meeting of the Acoustical Society of America, Austin, TX, April 9-12, 1985.
- (2) J. H. Ginsberg and J. S. DiMarco, "Finite Element Implementation of a Variational Principle for Acoustic Radiation and Scattering Exemplified by a Circular Disk," 110th Meeting of the Acoustical Society of America, Nashville, TN, November 4-8, 1985.
- (3) X. F. Wu, A. D. Pierce, and J. H. Ginsberg, "Application of a Variational Principle to Acoustic Radiation from a Vibrating Finite Cylinder," 111th Meeting of the Acoustical Society of America, Cleveland, OH, May 12-16, 1986.
- (4) J. H. Ginsberg and A. D. Pierce, "A Variational Principle for Sound Radiation from Vibrating Bodies," Rept. no. ADRL-86-101 (1986).
- (5) X. F. Wu, A. D. Pierce, and J. H. Ginsberg, "Rayleigh-Ritz Computational Technique for a Variational Formulation of Surface Pressure-- Exemplified by Sound Radiation from a Flat Circular Disk in Rigid Transverse Vibration," *IEEE Journal of Ocean Engineering*, OE-12, (1986) 412-418.
- (6) X. F. Wu, "Variational Principles for Acoustic Radiation and Diffraction from Underwater Structures," Ph.D. Thesis, Georgia Institute of Technology, 1987.
- (7) X. F. Wu, "Faster Calculations of Sound Radiation from Vibrating Cylinders Using Variational Formulations," *ASME J. Vib., Acoust., Stress, & Rel. in Design*, 111 (1989) 101-107.
- (8) J. H. Ginsberg and K. Wu, "Extension of the Surface Variational Principle to Arbitrary Motion of Bodies of Revolution," 123rd Meeting

of the Acoustical Society of America, Salt Lake City, UT, May 11-15, 1992.

- (9) J. H. Ginsberg and A. D. Pierce, "Variational Principle for Harmonic Fluid-Structure Interaction Applied to an Elastic Disk," 111th Meeting of the Acoustical Society of America, Cleveland, OH, May 12-16, 1986.
- (10) J. H. Ginsberg and P. T. Chen, "Application of a Variational Principle for Fluid-Structure Interaction to the Analysis of the Response of an Elastic Disk in a Finite Baffle," 114th Meeting of the Acoustical Society of America, Miami, FL, Nov. 16-20, 1987.
- (11) P. T. Chen and J. H. Ginsberg, "Conventional and Variational Formulations of Fluid-Structure Interaction for an Elastic Plate in a Baffle," Proceedings of Noise-Con 88, Purdue University, Indiana, June 20-22, 1988, 293-298.
- (12) P. T. Chen and J. H. Ginsberg, "Application of Variational Principles to the Evaluation of Axisymmetric Surface Pressure and Displacement along a Harmonically Vibrating Elastic Shell," Joint Meeting of the Acoustical Society of America and the Acoustical Society of Japan, Honolulu, Nov. 14-18, 1988.
- (13) P. T. Chen and J. H. Ginsberg, "Forced Vibration of a Submerged Spheroidal Shell Using Variational Principles," 117th Meeting of the Acoustical Society of America, Syracuse, NY, May 22-26, 1989.
- (14) J. McDaniel and J. H. Ginsberg, "Analysis by Variational Principles of the Vibration and Acoustic Radiation of a Vibrating Elastic Plate with Circumferential Stiffeners," 118th Meeting of the Acoustical Society of America, St. Louis, MO, November 27-December 1, 1989.
- (15) J. H. Ginsberg, P. T. Chen, and A. D. Pierce, "Analysis Using Variational Principles of the Surface Pressure and Displacement along

- an Axisymmetrically Excited Elastic Disk in a Baffle," *Journal of the Acoustical Society of America*, 88 (1990) 548-559.
- (16) "The Effect of Mode Truncation in an Analysis of Acoustic Radiation from Spheroidal Shells," P. T. Chen and J. H. Ginsberg, 119th Meeting of the Acoustical Society of America, Pennsylvania State University, May 21-25, 1990.
- (17) J. H. Ginsberg and J. G. McDaniel, "An Acoustical Variational Principle and Component Mode Synthesis Applied to the Analysis of Acoustic Radiation from a Concentrically Stiffened Plate," *ASME Journal of Vibration and Acoustics*, 113 (1991) 401-408.
- (18) J. H. Ginsberg and P. T. Chu, "Surface Variational Principle Analysis of the Response of an Eccentrically Loaded Submerged Disk in a Baffle," 121st Meeting of the Acoustical Society of America, Baltimore, MD, Apr. 29-May 3, 1991.
- (19) P. T. Chu, "Generalization of the Surface Variational Principle to Asymmetric Vibration of a Submerged Circular Plate," M. S. Thesis, Georgia Institute of Technology, 1991
- (20) J. H. Ginsberg and P. T. Chu, "Asymmetric Vibration of a Heavily Fluid-Loaded Circular Plate Using Variational Principles," *Journal of the Acoustical Society of America*, 91 (1992) 894-906.
- (21) P. T. Chen, "Axisymmetric Vibration, Acoustic Radiation, and the Influence of Eigenvalue Veering Phenomena in Prolate Spheroidal Shells Using Variational Principles," Ph.D. Thesis, Georgia Institute of Technology, 1992
- (22) P. T. Chen and J. H. Ginsberg, "Variational Formulation of Acoustic Radiation from Submerged Spheroidal Shells," *Journal of the Acoustical Society of America* 94, 221-233 (1993).



- (23) K. Wu, "Application of SVP to Scattering of Acoustic Waves at Arbitrary Incidence on Rigid and Elastic Bodies of Revolution," Ph.D. Thesis Proposal, Georgia Institute of Technology, Nov. 1993.
- (24) H. G. Kil and J. H. Ginsberg, "On the Extension of a Variational Principle for Surface Pressure to Treat Spurious Interior Cavity Resonances," 119th Meeting of the Acoustical Society of America, Pennsylvania State University, May 21-25, 1990.
- (25) X. F. Wu and A. D. Pierce, "Uniqueness of Solutions to Variationally Formulated Radiation Problems," ASME J. Vib. Acoustics 112 (1990) 263-267.
- (26) P. T. Chen and J. H. Ginsberg, "Axisymmetric Vibration of Spheroidal Shells for High Aspect Ratio," 118th Meeting of the Acoustical Society of America, St. Louis, MO, November 27-December 1, 1989.
- (27) P. T. Chen and J. H. Ginsberg, "Axisymmetric Vibrations of Spheroidal Shells: Eigenvalue Loci Veering and Parameter Sensitivity of Eigenfunctions," 120th Meeting of the Acoustical Society of America, San Diego, CA, Nov. 26-30, 1990.
- (28) P. T. Chen and J. H. Ginsberg, "Behavior of Multiple Degree of Freedom Systems Having Neighboring Eigenvalues: Veering of the Eigenvalue Loci and High Mode Sensitivity," 120th Meeting of the Acoustical Society of America, Baltimore, MD, Apr. 29-May 3, 1991.
- (29) J. H. Ginsberg and H. Pham, "Mode Localization in a Fluid-Loaded Plate," 122nd Meeting of the Acoustical Society of America, Houston, TX, Nov. 4-8, 1991.
- (30) "The Effect of Heavy Fluid Loading on Eigenvalue Loci Veering and Mode Localization Phenomena," Second International Congress on Recent Developments in Air- and Structure-Borne Sound and Vibration, Auburn University, March 4-6, 1992.

- (31) P. T. Chen and J. H. Ginsberg, "On the Relationship Between Veering of Eigenvalue Loci and Parameter Sensitivity of Eigenfunctions", *ASME Journal of Vibration and Acoustics*, 114 (1992) 141-148.
  
- (32) P. T. Chen and J. H. Ginsberg, "Modal Properties and Eigenvalue Veering Phenomena in the Axisymmetric Vibration of Spheroidal Shells," *Journal of the Acoustical Society of America*, 92, 1499-1508 (1992).
  
- (33) J. H. Ginsberg and H. Pham, "Forced Harmonic Response of a Continuous System Displaying Eigenvalue Veering Phenomena," *Structural Dynamics of Large Scale and Complex Structures*, Proceedings of 14th ASME Biennial Conference on Mechanical Vibration and Noise, DE-59 (1993) 59-66.

## **VI. PROJECT PARTICIPANTS**

1. Jerry H. Ginsberg, Principal Investigator, Professor and Woodruff Chair, School of Mechanical Engineering, Georgia Institute of Technology.
2. Allan D. Pierce, Co-Principal Investigator, Professor, School of Mechanical Engineering, Georgia Institute of Technology. Now Chairman, Dept. of Mechanical Engineering, Boston University.
3. Xiao Feng Wu, Graduate Research Assistant. Now Assistant Professor, Dept. of Mechanical Engineering, Wayne State University.
4. John DiMarco, Graduate Research Assistant. Now Research Engineer, Georgia Tech Research Institute.
5. Pei Tai Chen, Graduate Research Assistant. Now Associate Professor of Naval Architecture, National Taiwan Ocean University, Taiwan, ROC.
6. Pearl Chu, Graduate Research Assistant. Now Staff Engineer, Schlumberger-Doll, Houston, TX.
7. Kuangchung Wu, Graduate Research Assistant.
8. Hoang Pham, Graduate Research Assistant.